

# Phase noise due to vibrations in Mach-Zehnder atom interferometers

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## Abstract. –

Atom interferometers are very sensitive to accelerations and rotations. This property, which has some very interesting applications, induces a deleterious phase noise due to the seismic noise of the laboratory and this phase noise is sufficiently large to reduce the fringe visibility in many experiments. We develop a model calculation of this phase noise in the case of Mach-Zehnder atom interferometers and we apply this model to our thermal lithium interferometer. We are able to explain the observed phase noise which has been detected through the rapid dependence of the fringe visibility with the diffraction order. We think that the dynamical model developed in the present paper should be very useful to reduce the vibration induced phase noise in atom interferometers, making many new experiments feasible.

*Introduction.* – Atom interferometers are very sensitive to inertial effects [1,2] and this property was used to build accelerometers [3–10] and gyroscopes [11–15]. Because of this large sensitivity, a high mechanical stability of the experiment is required and several experiments [6,7,17,18] used an active control of the interferometer vibrations.

In the present letter, we study the phase noise induced by mechanical vibrations in three-gratings Mach-Zehnder thermal atom interferometers and we show that the rapid decrease of the fringe visibility with the diffraction order is largely due to this phase noise. Vibrations displace and bend the rail which holds the three diffraction gratings and we have developed a model based on elasticity theory to describe this dynamics. We are thus able to understand the contributions of various frequencies and to make a detailed evaluation of this phase noise in the case of our setup: the result is in good agreement with the phase noise value deduced from fringe visibility measurements. We have built a very stiff rail for our atom interferometer and this arrangement has revealed very efficient: the remaining phase noise is dominated by rotations of the rail, which should be reduced by a better rail suspension.

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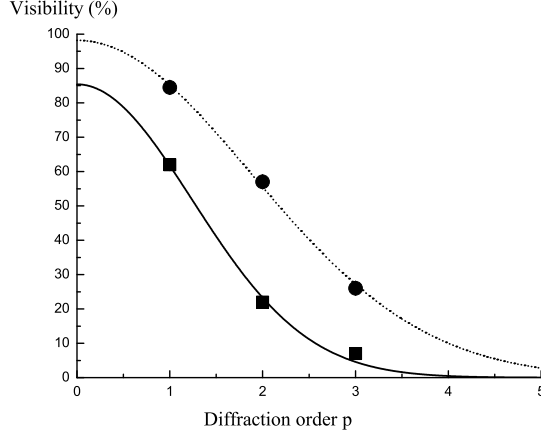


Fig. 1 – Fringe visibility as a function of the diffraction order  $p$ . Our measurements (round dots) are fitted by equation 2 with  $\mathcal{V}_{max} = 98 \pm 1$  % and  $\langle \Phi_1^2 \rangle = 0.286 \pm 0.008$ . The data points of Giltner and Siu Au Lee (squares) are also fitted by equation 2 with  $\mathcal{V}_{max} = 85 \pm 2$  % and  $\langle \Phi_1^2 \rangle = 0.650 \pm 0.074$ .

*Fringe visibility as a test of phase noise in atom interferometers.* – A phase noise  $\Phi(t)$  has the effect of reducing the fringe visibility  $\mathcal{V} = (I_{max} - I_{min}) / (I_{max} + I_{min})$ . Assuming a Gaussian distribution of  $\Phi$ , the visibility is given by [19,20]

$$\mathcal{V} = \mathcal{V}_{max} \exp [-\langle \Phi^2 \rangle / 2] \quad (1)$$

When the phase noise is due to inertial effects, we prove below that it is proportional to the diffraction order  $p$ ,  $\Phi_p = p\Phi_1$ , where  $\Phi_p$  corresponds to the order  $p$ . Equation (1) predicts a Gaussian dependence of the fringe visibility  $\mathcal{V}$  with the diffraction order  $p$  [20]:

$$\mathcal{V} = \mathcal{V}_{max} \exp [-p^2 \langle \Phi_1^2 \rangle / 2] \quad (2)$$

Only two atom interferometers have been operated with several diffraction orders, by Siu Au Lee and co-workers [18,21] in 1995 and more recently by our group [22]. The observed fringe visibility is plotted as a function of the diffraction order  $p$  in figure 1. A Gaussian fit, following equation (2), represents very well the data in both cases and the quality of this fit suggests the importance of a phase noise from inertial origin.

*Inertial sensitivity of Mach-Zehnder atom interferometers.* – We consider a three-grating Mach-Zehnder interferometer represented in figure 2. The inertial sensitivity of this type of interferometer is due to the existence of the diffraction phase which depends on the grating positions. The resulting phase of the interference signal is given by [19,20]:

$$\Phi_p = pk_G [2x_2(t_2) - x_1(t_1) - x_3(t_3)] \quad (3)$$

Here  $k_G = 2\pi/a$  is the grating wavevector ( $a$  being the grating period);  $p$  is the diffraction order and  $x_j(t_j)$  the  $x$ -coordinate of a reference point of grating  $G_j$  at time  $t_j$  when it is crossed by the atomic wavepacket. Equation (3) can be simplified by introducing the atom time of flight  $T = L_{12}/u$  from one grating to the next, with  $L_{12} = L_{23}$  and  $u$  being the atom

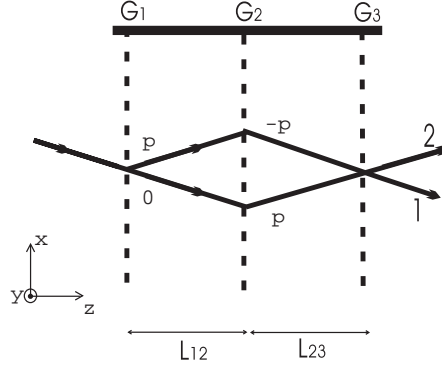


Fig. 2 – Schematic drawing of a three grating Mach-Zehnder atom interferometer, in the Bragg diffraction geometry. A collimated atomic beam is successively diffracted by three gratings  $G_1$ ,  $G_2$  and  $G_3$ . The diffraction orders corresponding to grating  $G_1$  and  $G_2$  are indicated on the two atomic beams. Two exit beams, labelled 1 and 2, carry complementary signals. The  $x$ ,  $y$ ,  $z$  axis are defined.

velocity (we will neglect its dispersion throughout the present paper). We can express the time  $t_j$  as a function of  $t_2$ , which will be noted  $t$ , and  $T$ :  $t_1 = t - T$  and  $t_3 = t + T$ . Then, if we expand  $\Phi$  in powers of  $T$  up second order, we get:

$$\Phi = pk_G \left[ \delta(t) - [v_{3x}(t) - v_{1x}(t)] T - \frac{[a_{1x}(t) + a_{3x}(t)] T^2}{2} \right] \quad (4)$$

Here  $\delta(t) = 2x_2(t) - x_1(t) - x_3(t)$  while  $v_{jx}(t)$  and  $a_{jx}(t)$  are the  $x$ -components of the velocity and acceleration of grating  $G_j$  measured by reference to a Galilean frame. In equation (4), the first term is due to the instantaneous bending  $\delta(t)$  of the rail, the second term represents Sagnac effect, as the velocity difference is related to the rail angular velocity, and the third term describes the sensitivity to accelerations.

*Theoretical analysis of the rail dynamics.* – We want to relate the positions  $x_j(t_j)$  of the three gratings to the mechanical properties of the rail and to its coupling to the seismic noise. As the interferometer is sensitive only to the grating  $x$ -coordinates, we use a 1D model to describe the rail dynamics. This model is based on elasticity theory [23] in order to describe, in an unified way, the motion and the deformations of the rail. In this model, the rail of length  $2L$  along the  $z$  direction can bend only in the  $x$  direction. The shape of its cross-section, assumed to be independent of the  $z$ -coordinate, is characterized by its area  $A = \int dx dy$  and by the moment  $I_y = \int x^2 dx dy$ , the  $x$ -origin being taken on the neutral line. The rail material has a density  $\rho$  and a Young's modulus  $E$ . The neutral line is described by a function  $X(z, t)$  which measures the position of this line with respect to a Galilean frame and which verifies [23]:

$$\rho A \frac{\partial^2 X}{\partial t^2} = -EI_y \frac{\partial^4 X}{\partial z^4} \quad (5)$$

The rail is coupled to the laboratory by forces and torques exerted at  $z = \epsilon L$  ( $\epsilon = \pm$ ) by its supports. The  $x$ -component of the force  $F_{x\epsilon}$  and the  $y$ -component of the torque are respectively related [23] to the third and second derivatives of  $X(z, t)$  with respect to  $z$ . We assume that the torques vanish so that  $\partial^2 X / \partial z^2 = 0$  at  $z = \epsilon L$  and that the force is the sum of an elastic term proportional to the relative displacement and a damping term proportional

to the relative velocity:

$$F_{x\epsilon} = -\epsilon EI_y \frac{\partial^3 X}{\partial z^3}(z = \epsilon L) = -K [X(\epsilon L, t) - x_\epsilon(t)] - \mu \frac{\partial [X(\epsilon L, t) - x_\epsilon(t)]}{\partial t} \quad (6)$$

where  $x_\epsilon(t)$  is the  $x$ -position of the support at  $z = \epsilon L$  and time  $t$ . We introduce the Fourier transforms  $X(z, \omega)$  and  $x_\epsilon(\omega)$  of the functions  $X(z, t)$  and  $x_\epsilon(t)$ . The general solution of equation (5) is:

$$X(z, \omega) = a \sin(\kappa z) + b \cos(\kappa z) + c \sinh(\kappa z) + d \cosh(\kappa z) \quad (7)$$

$a$ ,  $b$ ,  $c$  and  $d$  are the four  $\omega$ -dependent components of  $X(z, \omega)$ .  $c$  and  $d$  are related to  $a$  and  $b$ , thanks to assumption of vanishing torques. Then  $a$  and  $b$  are linearly related to the source terms  $x_\epsilon(\omega)$  by equations (6).  $\omega$  and  $\kappa$  are related by:

$$\rho A \omega^2 = EI_y \kappa^4 \quad (8)$$

When  $\mu$  is small enough, these equations predict a series of resonances. The first resonance, when  $\omega = \omega_{osc} = \sqrt{K/(\rho AL)}$ , describes an in-phase oscillation of the two ends of the rail while the second resonance, occurring when  $\omega = \omega_{osc}\sqrt{3}$ , describes a rotational oscillation of the rail around its center. Then, there is an infinite series of bending resonances occurring for  $\kappa_n$  verifying  $\cos(2\kappa_n L) \cosh(2\kappa_n L) = 1$  and  $\omega_n$  related to the  $\omega_0$ , by  $\omega_n = \omega_0(\kappa_n/\kappa_0)^2$ . In our model, the rail stiffness is described by one parameter only, namely  $\omega_0$ .

$$\omega_0 = 5.593 \sqrt{EI_y/(\rho AL^4)} \quad (9)$$

*The phase noise due to vibrations.* – The Fourier component  $\Phi_p(\omega)$  of the phase  $\Phi_p$  given by equation (3) can be expressed as a function of the amplitudes  $a$  and  $b$ . We assume that the grating reference points are on the neutral line, at  $z = \epsilon L_{12}$  ( $\epsilon = \pm$ ) and we get:

$$\begin{aligned} \Phi_p(\omega)/p = 2k_G \left[ b(\omega) \left( 1 - \cos(\kappa L_{12}) + (1 - \cosh(\kappa L_{12})) \frac{\cos(\kappa L)}{\cosh(\kappa L)} \right) \right. \\ + i a(\omega) \left( \sin(\kappa L_{12}) + \sinh(\kappa L_{12}) \frac{\sin(\kappa L)}{\sinh(\kappa L)} \right) \sin(\omega T) \\ \left. + b(\omega) \left( \cos(\kappa L_{12}) + \cosh(\kappa L_{12}) \frac{\cos(\kappa L)}{\cosh(\kappa L)} \right) (1 - \cos(\omega T)) \right] \quad (10) \end{aligned}$$

where the different lines correspond to the bending, the Sagnac and the acceleration terms in this order. This complicated equation can be given a very simple form by making expansions in powers of  $(\omega T)$  and  $\kappa L$  (assuming  $L_{12} = L$  for further simplification):

$$\begin{aligned} \Phi_p(\omega)/p \approx k_G \times \left[ [x_+(\omega) - x_-(\omega)] \frac{3i(\omega T)}{(3 - R)} \right. \\ \left. + [x_+(\omega) + x_-(\omega)] \frac{13.0(\omega/\omega_0)^2 + (\omega T)^2}{2(1 - R)} \right] \quad (11) \end{aligned}$$

where  $R = \omega^2 / [\omega_{osc}^2 - i(\omega_{osc}\omega/Q_{osc})]$ . Equation (11) has a limited validity because of numerous approximations but it gives a very clear view of the various contributions. The first term, proportional to  $[x_+(\omega) - x_-(\omega)]$  and to the time of flight  $T$ , describes the effect of the rotation of the rail excited by the out of phase motion of its two ends. This term, which is

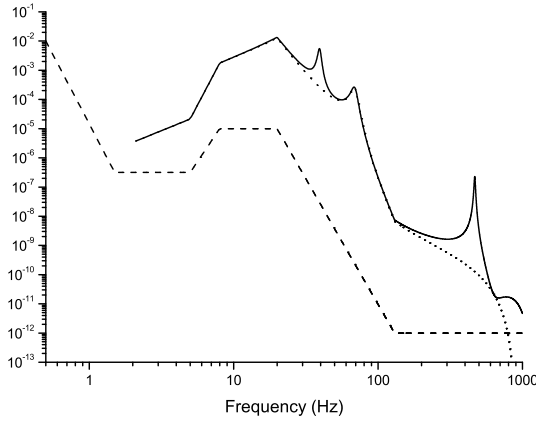


Fig. 3 – Calculated phase noise spectra  $|\Phi(\nu)/p|^2$  (full curve) and  $|\Phi_{Sagnac}(\nu)/p|^2$  (dotted curve), both in  $\text{rad}^2/\text{Hz}$  as a function of the frequency  $\nu$  in Hz. The smoothed seismic noise spectrum  $|x_\epsilon(\nu)|^2$  in  $\text{m}^2/\text{Hz}$  used in the calculation is plotted (dashed curve) after multiplication by  $10^{10}$ . The spectrum was recorded in the  $\nu = 0.5 - 100$  Hz range and assumed to be constant when  $10^2 < \nu < 10^3$  Hz.

independent of the stiffness of the rail, is sensitive to the rail suspension through the  $(3 - R)$  denominator. The second term is the sum of the bending term, in  $(\omega/\omega_0)^2$ , and the acceleration term, in  $(\omega T)^2$ . Both terms have the same sensitivity to the suspension of the rail, being sensitive to the first pendular resonance, when  $R \approx 1$ . The bending term is small if the rail is very stiff, i.e. when  $\omega_0$  is large.

*Application of the present analysis to our interferometer.* – When we built our interferometer, we knew that D. Pritchard [17, 19] and Siu Au Lee [18, 21] had been obliged to reduce  $\delta(t)$  in their atom interferometers by a servo-loop. Rather than using a servo-loop, we decided to improve the grating stability by building a very stiff rail. We use aluminium alloy for its large  $E/\rho$  ratio and we made the largest possible rail in the  $x$  direction to get a large  $I_y/A$  ratio. Using equation (9), we estimate  $\omega_0/2\pi \approx 437$  Hz, in reasonable agreement with our measurement,  $\omega_0/(2\pi) = 460.4$  Hz, with a rather large Q-factor,  $Q \approx 60$  (more details in [28]). The suspension of the rail is very simple, with rubber blocks made to support machine tools. From a rough estimate of their force constant  $K$  and the rail mass, the first resonance is calculated to be at  $\omega_{osc}/(2\pi) \approx 20$  Hz.

Following previous works [17–19, 24, 25], the instantaneous bending  $\delta(t)$  is conveniently measured by a 3-grating Mach-Zehnder optical interferometer attached to the gratings of the atom interferometer. We have built such an interferometer [26], with 200 lines/mm gratings from Paton Hawksley [27] ( $k_{g,opt} = 3.14 \times 10^5 \text{ m}^{-1}$ ) and an helium-neon laser at 633 nm. The phase  $\Phi_{opt}$  of the signal of such an optical interferometer is also given by equation (4) simplified because the time of flight  $T$  for light is negligible:  $\Phi_{opt} = pk_{g,opt}\delta(t)$ . In our experiment, the excitation of the rail by the environment gives very small signals, from which we deduce an upper limit of the bending  $\sqrt{\langle \delta(t)^2 \rangle} < 3$  nm.

To evaluate the phase noise, we need to know the seismic noise spectrum. A spectrum was recorded on our setup well before the operation of our interferometer and we use this measurement as a good estimate of the seismic noise. We have replaced the recorded spectrum

with several peaks appearing in the 8 – 60 Hz range by a smooth spectrum just larger than the measured one. Most of the peaks do not appear on a spectrum taken on the floor, because they are due to resonances of the structure supporting the vacuum pipes and their exact frequency has probably changed because of modifications of the experiment since the recording. The smoothed noise spectrum  $|x_\epsilon(\nu)|^2$  is plotted in figure 3. This figure also plots the calculated phase noise spectrum  $|\Phi(\nu)/p|^2$ , using equation (10), and the Sagnac phase noise spectrum  $|\Phi_{Sagnac}(\nu)/p|^2$  deduced from equation (10) by keeping only the term proportional to the  $a$  amplitude: clearly, Sagnac phase noise is dominant except near the in-phase pendular oscillation and the first bending resonance. The contribution of the in-phase pendular oscillation depends strongly on its frequency and  $Q$ -factor. The bending resonance is in a region where the excitation amplitude is very low, and, even after amplification by the resonance  $Q$ -factor, the contribution of the bending resonance to the total phase noise is fully negligible.

In this calculation, we have not used our estimate of the first pendular resonance  $\omega_{osc}/(2\pi) \approx 20$  Hz, because the predicted rms value of the bending  $\sqrt{\langle \delta(t)^2 \rangle}$  was considerably larger than the measured upper limit. We have used  $\omega_{osc}/(2\pi) = 40$  Hz, with  $Q_{osc} \approx 16$  and the measured  $\omega_0$  value,  $\omega_0/(2\pi) = 460.4$  Hz and  $T = 5.7 \times 10^{-4}$  s ( $L_{12} = 0.605$  m and  $u = 1065$  m/s). We assume that the two excitation terms  $x_\epsilon(\nu)$  have the same spectrum but no phase relation, so that we neglect the cross-term  $|x_+(\nu)x_-(\nu)|$ . For very low frequencies up to a few Hertz, we expect  $x_+(\nu) \approx x_-(\nu)$  and the associated correction would cancel the Sagnac term and this why we have not extended the  $|\Phi(\nu)/p|^2$  curves below 2 Hz. As soon as the frequency is larger than the lowest resonance frequency of the structure supporting the vacuum chambers (near 8 Hz), the assumption that  $x_+(\nu)$  and  $x_-(\nu)$  have no phase relation should be good.

By integrating the phase noise over the frequency from 2 to  $10^3$  Hz, we get an estimate of the quadratic mean of the phase noise:

$$\langle \Phi_p^2 \rangle = 0.16 p^2 \text{ rad}^2 \quad (12)$$

This estimate compares well with the value  $\langle \Phi_p^2 \rangle = (0.286 \pm 0.008) p^2$ , deduced from the fit of figure 1: we think that the agreement is convincing, if ones considers the large uncertainty on several parameters (seismic noise, frequency and  $Q$  factors of the pendular resonances), . This result is largely due to Sagnac phase noise, as the same integration only on Sagnac phase noise gives  $\langle \Phi_{Sagnac}^2 \rangle = 0.13 p^2 \text{ rad}^2$  and as shown by equation (11), this phase noise can be reduced only by a modification of the rail suspension.

*Conclusion.* – The present paper has analyzed the phase noise induced in a Mach-Zehnder atom interferometer by mechanical vibrations (more details in [28]). Starting from the well-known inertial sensitivity of atom interferometers, we have developed a simple 1D model describing the dynamics of the rail holding the diffraction gratings. This model provides an unified description of the low- and high-frequency dynamics, in which the rail behaves respectively as a solid object and an elastic object. In the low-frequency range, up to the frequency of the rotational resonance of the suspension, the out-of-phase vibrations of the two ends of the rail induce small rotations, which are converted into phase noise by Sagnac effect, and this is the dominant cause of inertial phase noise in our interferometer.

We think that the present analysis is important as it gives access to a reduction of this phase noise in atom interferometers. A better rail suspension should considerably reduce this phase noise. Then, we would be able to observe atom interference effects with an excellent fringe visibility, close to the fitted value  $\mathcal{V}_{max} = 98 \pm 1$  % of figure 1, and we would also be able to work either with diffraction orders  $p \gg 1$  or with considerably slower atoms.

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